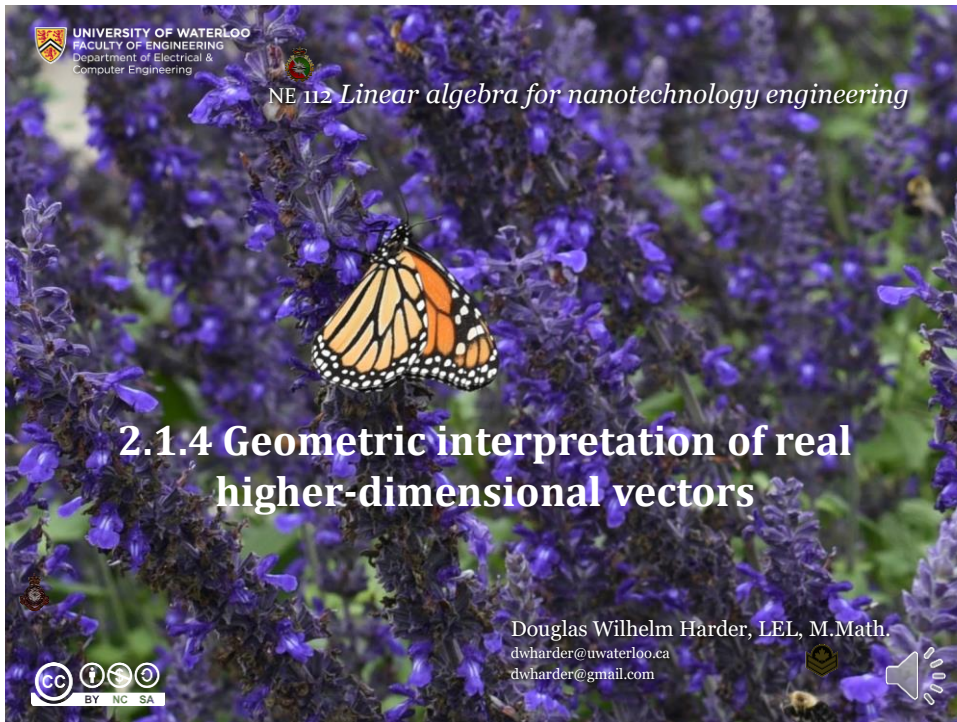




UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical & Computer Engineering

NE 112 *Linear algebra for nanotechnology engineering*

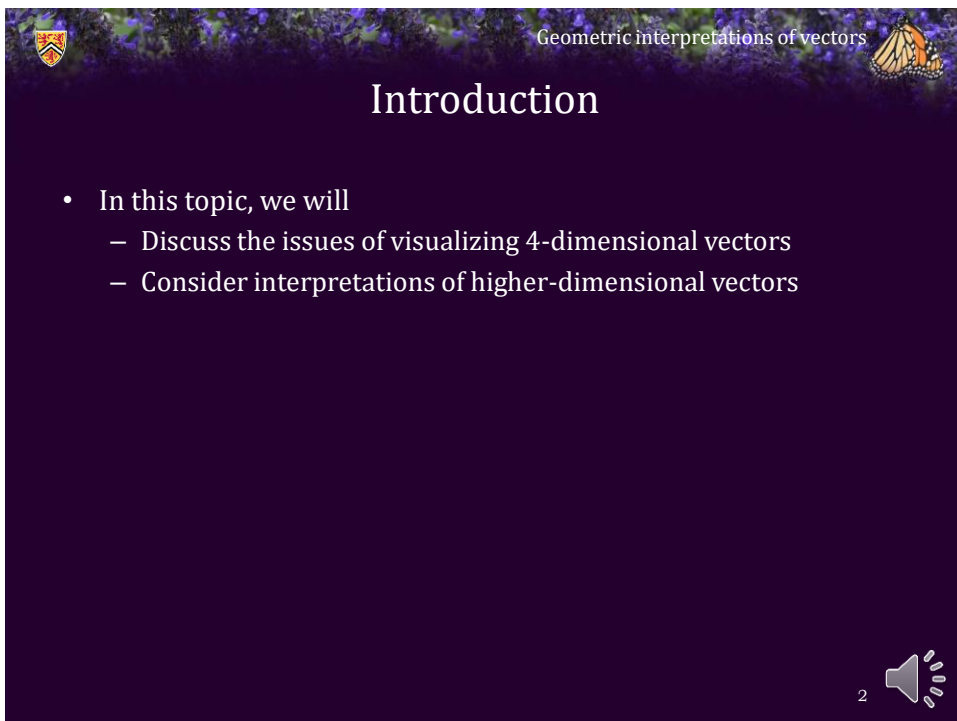
2.1.4 Geometric interpretation of real higher-dimensional vectors

Douglas Wilhelm Harder, LEL, M.Math.
dwharder@uwaterloo.ca
dwharder@gmail.com

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Speaker icon

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Geometric interpretations of vectors

Introduction

- In this topic, we will
 - Discuss the issues of visualizing 4-dimensional vectors
 - Consider interpretations of higher-dimensional vectors

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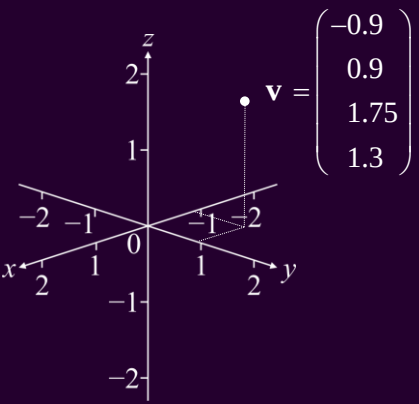
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Geometric interpretations of vectors

4-dimensional vectors

- It is interesting to interpret a 4th entry as time
 - This vector is 1.3 seconds into the future



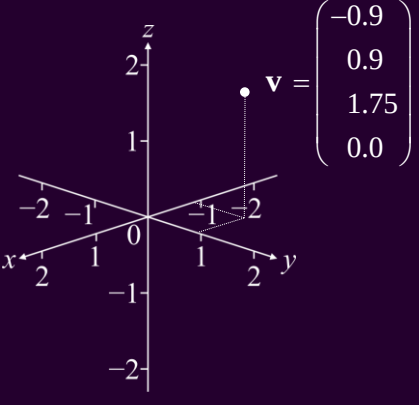
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Geometric interpretations of vectors

4-dimensional vectors

- It is interesting to interpret a 4th entry as time
 - This vector occupies this point in the present



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Geometric interpretations of vectors

n -dimensional vectors

- Instead, after 3 dimensions,
it is better to think of an n -dimensional vector
as a function of n integers

$$\mathbf{v} = \begin{pmatrix} -0.9 \\ 0.9 \\ 1.75 \\ 0.0 \end{pmatrix}$$

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Geometric interpretations of vectors

n -dimensional vectors

- If you must visualize such a point in 8 dimensions,
here is one such visualization

$$\mathbf{u} = \begin{pmatrix} -102 \\ 174 \\ -133 \\ 0 \\ 0 \\ 196 \\ -63 \\ 0 \end{pmatrix}$$

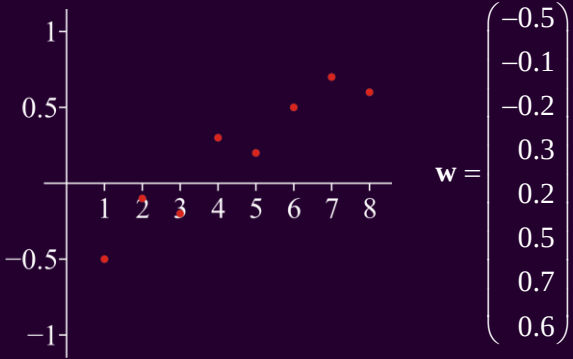
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Geometric interpretations of vectors

n -dimensional vectors

- If you must visualize such a point in 8 dimensions, here is one such visualization



$$\mathbf{w} = \begin{pmatrix} -0.5 \\ -0.1 \\ -0.2 \\ 0.3 \\ 0.2 \\ 0.5 \\ 0.7 \\ 0.6 \end{pmatrix}$$

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Geometric interpretations of vectors

n -dimensional vectors

- In general, however, it is best to think of vectors as ordered information
 - Don't spend too much time trying to *visualize* vectors
 - Each entry of the vector represents something in the real world

$$\mathbf{v} = \begin{pmatrix} -0.9 \\ 0.9 \\ 1.75 \\ 0.0 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} -102 \\ 174 \\ -133 \\ 0 \\ 0 \\ 196 \\ -63 \\ 0 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} -0.5 \\ -0.1 \\ -0.2 \\ 0.3 \\ 0.2 \\ 0.5 \\ 0.7 \\ 0.6 \end{pmatrix}$$

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Geometric interpretations of vectors

Summary

- Following this topic, you now
 - Seen interpretations of 4-dimensional vectors
 - Seen a possible interpretation of higher-dimensional vectors
 - Understand this is useful only if there is some sort of periodic relationship between the values
 - Temporal or spatial



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Geometric interpretations of vectors

References

[1] https://en.wikipedia.org/wiki/Linear_algebra



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Geometric interpretations of vectors



Acknowledgments

None so far.

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Geometric interpretations of vectors



Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see <https://www.rbg.ca/> for more information.





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Geometric interpretations of vectors

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